Finding the scale of space: derivation of the star distance formula

In the article 'Finding the scale of space', we calculated the distance of a 'star' in a classroom by using parallax distance measurements and a camera. The distance *d* of the star was calculated from the measured quantities using the following equation:

$$d = \frac{p_L \times d_L \times b}{L \times p}$$

where:

d = distance to star

L = actual length of the calibration object

b = actual distance the camera was moved (which corresponds to the distance from C_A to C_B)

d_L = actual distance of the calibration object from the camera baseline (along line OQ)

p = distance as the number of pixels between the star images (at D_A and D_B)

p_L= length as the number of pixels of the image of the calibration object

In fact, it is quite straightforward to derive this equation using the mathematical idea of similar triangles. The steps below explain how.

Supporting material for:

Pössel M (2017) Finding the scale of space. *Science in School* **40**: 40–45. www.scienceinschool.org/2017/issue40/parallax2

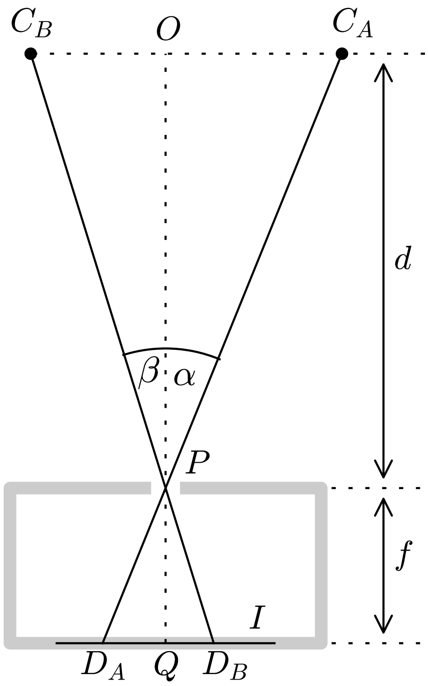

Figure 1: Simplified model of the parallax setup (Image courtesy of HdA / M Pössel)

1. Looking at the geometry in figure 1, we can see that the triangle C_BPC_A is similar to the triangle D_APD_B (since their corresponding angles are equal). So if we use *l* to represent the distance between the star positions D_A and D_B (in the image plane, I), from similarity it follows that:

$$d = \frac{f \times b}{l}$$

2. The distance *l* is proportional to the distance between the two star positions in our photographic image, expressed as a number of pixels, *p*. If we use *k* to represent the constant factor (yet to be determined) that relates the number of pixels to actual lengths in the image plane, and set $S = k \times f$, it follows that:

$$d = \frac{S \times b}{p}$$

3. We now apply the same reasoning to the calibration object, which we have placed parallel to the camera baseline at a distance d_L from it. This distance, which we measure directly, and the image length of the calibration object in pixels (p_L) are related by the equation below:

$$d_L = \frac{S \times L}{p_L}$$

4. We can eliminate *S* by combining the two equations above. First, we rearrange the equation in step 3 to isolate S, by multiplying both sides by p_L and dividing both sides by *L*:

$$S = \frac{d_L \times p_L}{L}$$

5. We now substitute this expression for *S* into the equation derived in step 2, which yields a formula linking the distance *d* to the other known lengths *b* and *f*.

The formula, as we have seen, is:

$$d = \frac{p_L \times d_L \times b}{L \times p}$$